

Homework #5 (10 points) - Show all work on the following problems:

Problem 1 (2 points): Derive the exact reflection and transmission coefficients R and T for normal incidence of light on an interface between two materials, without assuming that $\mu_1 = \mu_2 = \mu_0$. Express your answer in terms of $\beta = \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1}$. Explicitly show that R + T = 1.

Problem 2 (3 points): Construct a graph like the one in Fig. 9.16 for the case of an electromagnetic wave incident from vacuum into diamond, which has an index of refraction $n = 2.42$. Assume that $\mu_1 = \mu_2 = \mu_0$.

3a (0.5 points): Calculate the numerical values for the amplitudes E_{OR} and E_{OT} at normal incidence, using the convention of negative amplitude values to indicate if one of the waves is out of phase with the incident wave.

3b (0.5 points): Calculate Brewster's angle.

3c (1 point): Calculate the crossover angle, where $E_{OR} = E_{OT}$.

3d (1 point): Draw the graph!

Problem 3 (5 points): We have worked exclusively with plane wave solutions so far. However, for point sources of electromagnetic radiation, a more natural solution is a spherical wave. In this case, the real electric field can be written (with $\frac{\omega}{k} = c$):

$$\vec{E}(r, \theta, \phi, t) = A \frac{\sin\theta}{r} \left[\cos(kr - \omega t) - \frac{1}{kr} \sin(kr - \omega t) \right] \hat{\phi}$$

For notational convenience, you might wish to write this in the following shorthand form:

$$\vec{E}(r, \theta, \phi, t) = A \frac{\sin\theta}{r} \left[\cos u - \frac{1}{kr} \sin u \right] \hat{\phi}$$

If you write it like this, be careful to remember the r and t dependence of u when you take derivatives!

3a (2.5 points): Plug this electric field into Faraday's law, and integrate with respect to time to find the corresponding magnetic field.

3b (2.5 points): Calculate the magnitude and direction of the corresponding time-dependent Poynting vector, and then average it over a full cycle of the wave to find the average energy flux (which is the intensity I).